Remarks :-

① **Groupoid** :- Let G be a non-empty set on which a binary composition 'o' is defined. Some algebric structure is imposed on G by the composition 'o' and (G, o) becomes an algebric system. The algebric system (G, o) is said to be Groupoid.

For example :- $(\mathbb{Z}, +)$, $(\mathbb{Z}, -)$, $(\mathbb{R}, +)$ etc are groupoid.

② **semi-group** :- A groupoid (G, o) is said to be a semigroup if 'o' is associative.

i,e, a semi-group has two properties
(i) closure property.
(ii) Associative.

For example :- $(\mathbb{Z}, +)$, $(\mathbb{R}, +)$ etc are semi-groups. But $(\mathbb{Z}, -)$ be not a semigroup, since

$2-(5-6) \neq (2-5)-6$.

$\therefore$ '$-$' is not associative on the set $\mathbb{Z}$.

③ **Monoid** :- A semigroup (G, o) containing the identity element is said to be ~~moo~~ monoid.

i,e, Monoid has three properties
(i) closure property.
(ii) Associative property.

(iii) Identity element.

For example:- $(\mathbb{Z},+)$, $(\mathbb{Z},\cdot)$, $(\mathbb{R},+)$, $(\mathbb{R},\cdot)$ all are monoids. But $(E,\cdot)$ is not monoid since it does not contain identity element. (where E be the set of all even integers.)

8. Let (G,o) be a group and $c \in G$. Define a binary composition $*$ on G by $a*b = a o c o b$ for $a,b \in G$. Show that $(G,*)$ is a group with c^{-1} as the identity element.

<u>proof</u>:- Given that $(G,*)$ be a group and

$a*b = a o c o b$ where $a,b,c \in G$.

clearly $a*b \in G$ since 'o' is closed in G as (G,o) is a group.

clearly 'o' is associative on G, so $*$ is also associative on G.

Let e be the identity element of $(G,*)$ then $\forall a \in G$

$a*e = a$

$\Rightarrow a o c o e = a \quad \Rightarrow (a^{-1} o a) o c o e = a^{-1} o a$

~~$\Rightarrow e_G c o e = a^{-1} o$~~ $\quad \Rightarrow e_G o c o e = e_G$

e_G being identity element of (G,o)

$\Rightarrow c o e = e_G \Rightarrow e = c^{-1}$ [~~from left~~ cancell

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clearly identy element $c^{-1} \in G$.

now let y be inverse of x such that

$x * y = y * x = e = c^{-1}$

$\Rightarrow x \circ c \circ y = c^{-1}$ (where $c \in G, x \in G$)

$\Rightarrow y = c^{-1} \circ x^{-1} \circ c^{-1} \in G$

this shows that all elements of x has its own inverse in G.

. This shows that $(G, *)$ is a group with c^{-1} identity element.

<u>Lets do yourself</u> (similar to last problem in last class)

<u>Q</u> i) Let $(G, \circ)$ be a group and $a \in G$. Define a mapping $\lambda_a : G \to G$ by $\lambda_a(x) = a \circ x$; $x \in G$. prove that λ_a is a bijection. show that λ_e is the identiy element.

ii) Let $S = \{ \lambda_a : a \in G \}$. Define a binary composition $*$ on S by $\lambda_a * \lambda_b = \lambda_{(a \circ b)}$ for $\lambda_a, \lambda_b \in S$

Show that $(S, *)$ is a group.

<u>Sol^n</u>:- (i) Given that $(G, \circ)$ be a group. so, it bears four properties.

now clearly $\forall a, b \in G$; $a \circ b \in G$

$\therefore \lambda_{(a \circ b)} \in S$

$\therefore \lambda_a * \lambda_b = \lambda_{(a \circ b)} \in S \quad \forall a, b \in G$.

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$\therefore$ '$*$' is closed on S.

$\therefore$ closure property holds.

I clearly $a \circ (b \circ c) = (a \circ b) \circ c \quad \forall\, a, b, c \in G$.
since $(G, \circ)$ be a group

② now $(\lambda_a * \lambda_b) * \lambda_c$

$= \lambda_{(a \circ b)} * \lambda_c$

$= \lambda_{(a \circ b) \circ c} = \lambda_{a \circ (b \circ c)}$

$(\lambda_a * \lambda_b) * \lambda_c = \lambda_a * (\lambda_b * \lambda_c) \quad a, b, c \in G$.
this shows that '$*$' satisfies associative property on S.

II Let e being identity element in $(G, \circ)$
(since $(G, \circ)$ be a group so it contains identity element)

now
$\lambda_a * \lambda_e = \lambda_{(a \circ e)} = \lambda_a \quad \forall\, a \in G$.

and $\lambda_e * \lambda_a = \lambda_{(e \circ a)} = \lambda_a$

$\therefore \lambda_a * \lambda_e = \lambda_e * \lambda_a = \lambda_a \quad \forall\, a \in G$.
this shows that λ_e be identity element in S.

D since $(G, \circ)$ be a group so each element in G has its own inverse in G.
Now ~~let~~ let λ_a be an element of S also $\lambda_{(a^{-1})} \in S$ since $a, a^{-1} \in G$.

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now

$f_a * f_{a^{-1}} = f(a^{-1} * a) = f_e$ (e being identity element in G)

and $f_{a^{-1}} * f_a = f(a^{-1} * a) = f_e$.

this shows that $f_a * f_{a^{-1}} = f_{a^{-1}} * f_a = f_e$

$\therefore$ Inverse of element of S exists in S. $\forall a \in G$.

So, $(S, *)$ is a group.

Q. Let $(G, \circ)$ be a group and $a \in G$. Define $I_a : G \to G$ by $I_a(x) = a^{-1} \circ x \circ a$; $x \in G$. Prove that I_a is a bijection. What happens if $(G, \circ)$ be a commutative group?

proof:- $I_a(x) = a^{-1} \circ x \circ a$ where $a \in G$. $x \in G$.

now let $y \in G$

$\therefore I_a(y) = a^{-1} \circ y \circ a$

now $I_a(x) = I_a(y)$

$\Rightarrow a^{-1} \circ x \circ a = a^{-1} \circ y \circ a$

$\Rightarrow x \circ a = y \circ a$ [By left cancellation law]

$\Rightarrow x = y$ [By right cancellation law].

$\therefore x \neq y \Rightarrow I_a(x) = I_a(y)$; this shows that I_a is injective.

Let p be an arbitrary element of

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the codomain set G. since $a \in G$ and $p \in G$ so there exist a unique element x in G such that $a^{-1} \circ x \circ a = p$ holds.

$\therefore$ x is the pre-image of p, so I_a is surjective.

Since I_a is injective as well as surjective, so it is bijective.

If $(G, \circ)$ is commutative so $\forall\, x, y \in G$

$x \circ y = y \circ x$ holds

now $I_a(x) = a^{-1} \circ x \circ a$; $a, x \in G$

$= a^{-1} \circ a \circ x$ [Since $(G, \circ)$ is Commutative group]

$I_a(x) = e \circ x$

Where e being identity element in $(G, \circ)$

$\boxed{I_a(x) = x}$

this shows that $I_a(x)$ becomes then an identity mapping.

So if $(G, \circ)$ be a commutative group then $I_a : G \to G$ becomes an identity mapping.

(Where $I_a(x) = a^{-1} \circ x \circ a$; $x, a \in G$)

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Definition and examples of groups, examples of abelian and non-abelian groups, the group Z_n ofintegers under addition modulo n and the group U(n) of units under multiplication modulo n.Cyclic groups from number systems, complex roots of unity, circle group, the general lineargroup $GL_n(n,R)$, groups of

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symmetries of (i) an isosceles triangle, (ii) an equilateral triangle,(iii) a rectangle, and (iv) a square, the permutation group Sym (n), Group of quaternions.

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